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A NONPARAMETRIC TEST FOR EXTINCTION BASED ON A SIGHTING RECORD

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Abstract. Extinctions of plant and animal species are rarely observed directly and must be inferred from sighting or collection records. The inference about extinction that can be drawn from a sighting record depends on the sighting rate. Existing methods assume that the form of the sighting rate is known and may be sensitive to deviations from the assumed form. A simple nonparametric test is proposed that makes minimal assumptions about the sighting rate. The test is illustrated by application to collection records of nine Mauritian orchid species.

Key words: extinction, testing for; extinction inference based on sighting records; orchids and extinction; Poisson process, nonstationary; sighting records, use in inferring extinction; truncation point.

INTRODUCTION

Extinctions of particular plant and animal species are rarely observed directly and, in many cases, can only be inferred from sighting or collection records. Statistical methods for inference about extinction based on a sighting record have been discussed by Solow (1993*a*, *b*), Burgman et al. (1995), McCarthy (1998), and others. The basic idea underlying these methods is that confidence in the continued existence of a species is greater the more recently it has been sighted. To use this idea as the basis for a formal test of the null hypothesis that a species is extant, it is necessary to understand the sampling distribution of the sighting record under the null hypothesis. This distribution is sensitive to variations over the period of observation in the sighting rate. Such variations can arise, for example, from changes in the abundance of the species or in the sighting effort.

While it is possible, in principle, to devise a test when the form of the sighting rate is known, this knowledge is commonly unavailable. This paper describes a simple method that can be used in this situation. The method is based on an early idea of Robson and Whitlock (1964) for inference about a truncation point of a probability distribution. In related work, Solow (2002) used this work to construct a confidence interval for the lower endpoint of the stratigraphic range of fossil taxa.

STATISTICAL MODEL

Suppose that during the observation period $(0, T)$ sightings of a species occur at ordered times $t_1 < t_2 < \ldots < t_n$. These sighting times are assumed to follow a (possibly nonstationary) Poisson process with unknown sighting rate:

$$\begin{aligned} \lambda(t) &> 0 \qquad 0 \leq t \leq T_e \\ &= 0 \qquad t > T_e \end{aligned} \tag{1}$$

where T_e is the unknown extinction time. Interest centers on testing the null hypothesis H_0: $T_e = T$ (or, equivalently, $T_e > T$) that the species remains extant at time T against the alternative hypothesis H_1: $T_e < T$ that the species is extinct.

It is straightforward to show that, conditional on their number n, the ordered sighting times have the same distribution as an ordered sample of size n from the probability distribution with probability density function (pdf)

$$f(t) = \frac{\lambda(t)}{\int_0^{T_e} \lambda(u)\, du} \qquad 0 \leq t \leq T_e \tag{2}$$

(Cox and Lewis 1978). For example, if the rate function is constant prior to T_e, then $f(t)$ is the uniform pdf on the interval $(0, T_e)$. More generally, the pdf in Eq. 2 is said to be truncated at the right at T_e. Thus, conditional on the number of sightings, the inference problem for the extinction of a Poisson process is equivalent to the inference problem for the endpoint of a truncated distribution.

METHODS

Solow (1993*a*) described a test for extinction in the case where, under H_0, the sighting rate is assumed to be constant over the observation period. For this model,

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TABLE 1. Estimated true probability of false rejection of H_0 for the test for extinction of Solow (1993*a*) with nominal significance level 0.10 when sighting times follow truncated exponential and reversed truncated exponential distributions over the unit interval for selected values of n and θ.

		P values	
n	θ	Truncated exponential	Reversed truncated exponential
10	1.0	0.243	0.034
	2.0	0.436	0.007
30	1.0	0.255	0.023
	2.0	0.481	0.004

Notes: H_0 = the species is extant at time T; n = the number of sightings of the species; θ = exponential parameter. Results are based on simulating 10 000 samples of site n from the corresponding distribution.

$f(t)$ is uniform over the interval $(0, T_e)$. Under this assumption, the observed significance level or P value in testing H_0 against H_1 is $(t_n/T)^n$. The null hypothesis H_0 is rejected in favor of H_1 at a specified significance level α if this P value is less than α. When the beginning of the observation period is unknown, it can be taken as the time of the first sighting and the analysis is restricted to the remaining $n - 1$ sightings. To show the sensitivity of this method to a failure of the assumption of a constant sighting rate, the true probability of false rejection of H_0 when the nominal value of α is 0.10 is given in Table 1 for the case where $T_e = T = 1$ and the sighting rate declines according to $\lambda(t) \propto \exp(-\theta t)$. For this model, $f(t)$ is the exponential distribution with mean $1/\theta$ truncated on the right at 1. Results are shown for selected values of θ and n. Each of the values in Table 1 was based on simulating 10 000 samples of size n from the corresponding distribution. Turning to Table 1, it is clear that the decline in the sighting rate leads the test to detect extinction falsely with probability far greater than the nominal 0.10 level. The reason is that the declining sighting rate gives rise to a most recent sighting that tends to occur earlier in the observation period than expected under a constant sighting rate. The practical consequence is that the support that the sighting record lends to the hypothesis that the species is extinct is considerably weaker than the test suggests.

To illustrate the effect on this test of an increasing sighting rate, the simulation experiment was repeated using the reversed truncated exponential distribution—that is, with sighting times simulated as $1 - X$ where X has an exponential distribution with mean $1/\theta$ truncated on the right at 1. Results are also given in Table 1. In this case, the probability of falsely detecting extinction is far below the nominal value of 0.10. Here, the increasing sighting rate gives rise to a most recent sighting that tends to occur later in the observation period than expected under a constant sighting rate. In this case, the support that the sighting record lends to the hypothesis that the species is extinct is considerably greater than the test suggests.

As noted, if a parametric model for $\lambda(t)$ is available, then it may be possible to devise a valid test for extinction. For example, Solow (1993*b*) described such a test for the case considered above where the sighting rate declines exponentially at an unknown rate. For later use, the P value for this test is given by $F(t_n)/F(T)$ where

$$F(t) = 1 - \sum_{i=1}^{[s/t]} (-1)^{i-1} \binom{n}{i} \left(\frac{1 - it}{s}\right)^{n-1} \qquad (3)$$

where $s = \Sigma_{i=1}^{n} t_i$ and [ ] denotes the integer part. Simulation results not reported here show that, as expected, the true probability with which the test falsely rejects H_o is well below the nominal significance level if the sighting rate is constant. This discrepancy is even greater when the sighting rate is proportional to a reversed truncated exponential distribution.

It is not always straightforward to find a test for a given parametric form for $\lambda(t)$. Even if a parametric test can be devised, as a practical matter it can be difficult to specify a parametric model for $\lambda(t)$ when n is small. Moreover, as with the tests based on constant or exponential sighting rates, the results may be sensitive to deviations from the assumed model. Alternative methods can be based on asymptotic considerations whereby the most recent sightings are treated as an ordered sample from the limiting Weibull extreme value distribution (e.g., Smith and Weissman 1985, Hall and Wang 1999). However, these methods are rather complicated and do not work well, in the sense that H_o cannot be rejected no matter how great the interval between the most recent sighting and the end of the observation period, for sample sizes typical of sighting records. Here we describe a simple, nonparametric method for testing for extinction based on the work of Robson and Whitlock (1964) on estimating a truncation point. In the present context, their main result is that an approximate $1 - \alpha$ confidence interval for T_e is given by $(T_n, T_n + 1 - \alpha/\alpha\ (T_n - T_{n-1}))$. It follows from the connection between confidence intervals and hypothesis tests that an approximate P value in testing H_0 against H_1 is simply the ratio

$$p = \left(\frac{T_n - T_{n-1}}{T - T_{n-1}}\right) \qquad (4)$$

of the interval between the second most recent and most recent sightings to the interval between the second most recent sighting and the end of the observation period. This test leads to simple rules of thumb. For example, H_0 can be rejected at conventional significance levels if this ratio exceeds 20.

TABLE 2. Estimated true probability of false rejection of H_0 for the nonparametric test for extinction. Format and symbols are as in Table 1.

		P values	
n	θ	Truncated exponential	Reversed truncated exponential
10	1.0	0.111	0.092
	2.0	0.133	0.088
30	1.0	0.105	0.095
	2.0	0.109	0.100

TABLE 3. Estimated power of the parametric test for extinction of Solow (1993*a*) and the nonparametric test when the sighting rate is constant for selected values of *n* (the number of sightings) and T_e (the time of extinction). The observation period is the unit interval, and both tests were applied at the 0.10 significance level.

n	T_e	Parametric test	Nonparametric test
10	0.5	1.000	0.720
	0.8	0.929	0.327
30	0.5	1.000	0.973
	0.8	1.000	0.612

In essence, this test bases the assessment of significance on the behavior of the sighting record in the vicinity of the most recent sighting. In particular, there is no need to specify either the beginning of the observation period or even the number of sightings in the observation period. This can be a distinct practical advantage. While parametric tests use all of the information in the sighting record, to do so they require complete knowledge of the observation period and the number of sightings in it and the specification of a model of the sighting rate.

The nonparametric *P* value given in Eq. 4 is exact if $\lambda(t)$ is constant and asymptotically correct if $\lambda(t)$ is positive and continuous at T_e. To assess its validity when *n* is small, it was applied to data simulated from the truncated exponential and reversed truncated exponential distributions as outlined above. The results are presented in Table 2. In all cases, the true probability of false rejection of H_0 is reasonably close to the 0.10 nominal level. There is a clear tendency for this probability to exceed 0.10 in sampling from the truncated exponential distribution and to fall below 0.10 in sampling from the reversed truncated exponential distribution. However, the discrepancies are small in comparison to those in Table 1 and decline with increasing *n*.

The robustness of validity of the nonparametric test is expected to come at the cost of lower power than the correct parametric test. Recall that power refers to the probability of rejecting H_0 when it is false. In Table 3, the power of the nonparametric test is compared to the power of the test of Solow (1993*a*) in the case where the sighting rate is constant over the interval $(0, T_e)$ for selected values of *n* and T_e with $T = 1$ and $\alpha = 0.10$. As before, each entry in this table was based on 10 000 simulated samples of size *n*. Note that the test of Solow (1993*a*) has power 1 provided $T_e \leq T\alpha^{1/n}$. This occurs in three of the four cases reported in Table 3. The lower power of the nonparametric test is clear, although it is also clear that the loss in power can be small when *n* is not too small and extinction occurs not too late in the observation period. In Table 4, the power of the nonparametric test is compared to the power of the test of Solow (1993*b*) in the case where the sighting rate declines exponentially at rate θ over the interval $(0, T_e)$ for selected values of *n*, θ, and T_e, again with $T = 1$ and $\alpha = 0.10$. In this case, as a result of its elevated probability of false rejection of H_0, the power of the nonparametric test actually exceeds the power of the parametric test in some cases.

AN APPLICATION TO MAURITIAN ORCHIDS

In this section, we apply the test outlined above to collection records of Mauritian orchid species. A total of 89 orchid species have been identified on Mauritius, of which 9 species are endemic. Stahm and Bosser (1996) concluded that 24 of these species had been extirpated or had become extinct by 1996 and that the continued existence of several more was in doubt. Beginning in 1997, a four-year study was conducted of the Mauritian orchid flora. Fieldwork conducted as part of this study located several species that had been thought to be extinct or whose status had been in doubt. The study also involved a survey of herbarium materials, resulting in the collection records analyzed here.

Here, we focus on the nine species with sighting records that contained at least eight sightings prior to

TABLE 4. Estimated power of the parametric test for extinction of Solow (1993*b*) and the nonparametric test when the sighting rate declines exponentially at rate θ for selected values of *n*, θ, and T_e. The observation period is the unit interval, and both tests were applied at the 0.10 significance level.

n	θ	T_e	Parametric test	Nonparametric test
10	1.0	0.5	0.590	0.634
		0.8	0.337	0.266
	2.0	0.5	0.470	0.621
		0.8	0.217	0.242
30	1.0	0.5	0.996	0.941
		0.8	0.914	0.507
	2.0	0.5	0.965	0.914
		0.8	0.632	0.401

Note: T_e = the time of species extinction; *n* = the number of sightings of the species.

TABLE 5. Significance levels for tests for extinction based on sighting records; comparison of the parametric tests of Solow (1993*a, b*) and the nonparametric test, for nine Mauritian orchid species.

Species	*n*	*P* values		
		Solow (1993*a*)	Solow (1993*b*)	Nonparametric
Angraecum aff. rutenbergianum	11	0.035	0.000	0.100
Angraecum calceolus	15	0.003	0.004	0.136
Angraecum mauritianum	8	0.078	0.003	0.043
Angraecum ramosum	9	0.092	0.940	0.190
Benthamia spiralis	8	0.175	0.105	0.118
Bulbophyllum caespitosum	9	0.124	0.006	0.091
Calanthe candida	8	0.806	0.931	0.950
Calanthe sylvatica	8	0.036	0.108	0.056
Polystachya concreta	13	0.019	0.006	0.200

1997 and that were, in the course of the study, found to be extant. The significance levels from this test are reported in Table 5. As an illustration of the underlying calculation, the record for *Angraecum aff. rutenbergianum* contains $n = 11$ collections in 1933, 1941, 1952, 1962, 1965, 1966, 1967, 1971, 1974, 1976, and 1978. The interval between the two most recent sightings is two years and the interval between the second most recent sighting and the end of the observation period (1996) is 20 years. The significance level given by the ratio of these intervals is 0.10. For the purposes of comparison, significance levels for the test of Solow (1993*a*) that assumes a constant pre-extinction sighting rate and the test of Solow (1993*b*) that assumes an exponentially declining pre-extinction sighting rate are also reported. If the test is applied at the 0.05 significance level, then the nonparametric test incorrectly rejects H_0 in 1 of 9 cases. The corresponding figure for the tests of Solow (1993*a*) and Solow (1993*b*) are 4 of 9 and 5 of 9, respectively.

DISCUSSION

Inference about extinction based on a sighting record remains a difficult problem. The main difficulty is that the inference that can drawn from a sighting record depends on how the sighting rate varies. Existing methods assume that the form of this rate is known and, as illustrated here, can be very sensitive to errors in the assumed form. While the nonparametric test proposed in this paper makes minimal assumptions about this rate, it does assume that, under the null hypothesis, the sighting rate is smooth (and positive) at the end of the observation period. An important potential source of variation in the sighting rate is variation in sighting effort. If sightings arise from regular surveys, such as Christmas bird lists, or from accidental encounters, then the assumption underlying this test may be reasonable. However, if sightings arise from sporadic expeditions separated by periods of no effort, then this assumption may not be reasonable. This underlines the general point that the interpretation of sighting data—as with other kinds of ecological data—requires an understanding of the process by which the data were generated.

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